Towards a Mechanized Proof of Selene Receipt-Freeness and Vote-Privacy

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“Security protocols are three line programs that people still manage to get wrong”

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Voting protocols are ten line programs that:

- use **hard crypto**
  homomorphic encryption, zero-knowledge proofs, commitment schemes, oblivious transfers, threshold cryptography...

- have **colorful security properties**
  vote-privacy, individual verifiability, universal verifiability, coercion resistance, receipt freeness...

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- *sometimes* do not come with **security proofs**

- (and kill your favourite verification engine)
Contributions

We propose:

- The first formal model of the Selene voting protocol
- A simplified version of the protocol amenable to automatic verification
- A convergent equational theory for Pedersen-style commitments used by Selene

Results:

- We prove Vote Privacy in our model
- We show a known attack for Selene Receipt Freeness, and prove the security of the corrected version
The Selene E-voting Protocol

- Due to Peter Ryan, Peter Rønne, Vincenzo Iovino
- Internet voting protocol designed for low-coercion scenarios
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Key ideas

1. votes are publicly posted on a bulletin board makes it easy to trust the result;
2. tracking receipts (tracker numbers) allow users to trust that their vote has been cast, ✓ individual verifiability
3. and to fake receipts for potential coercers. ✓ receipt freeness
**El-gamal cryptosystem**

**Gen:** Select a subgroup $G \subset \mathbb{Z}_p^*$ of order $q$, and a generator $g$ of $G$. Choose $x \leftarrow \mathbb{R} \mathbb{Z}_q$. Reveal $h = g^x$.

**Enc:** To encrypt a message $m \in G$, we choose $r \leftarrow \mathbb{R} \mathbb{Z}_q$. The ciphertext is then:

$$(c, d) = (g^r, m \cdot h^r).$$

**Dec:** To decrypt the ciphertext $(c, d)$, compute

$$m = \frac{d}{c^x}.$$
El-gamal homomorphisms:

Reencryption:

Let \((g^r, m \cdot h^r)\) be an encryption of \(m\) with randomness \(r\).

By choosing \(r' \leftarrow \mathbb{Z}_q\), we can re-encrypt the message \(m\) with
\[
(g^{r+r'}, m \cdot h^{r+r'}) = (g^r, m \cdot h^r) \cdot (g^{r'}, 1 \cdot h^{r'})
\]

Shuffling mixnets can be built by chaining re-encryption mixers, that apply re-encryption and randomly shuffle the values.

If at least one node in the mixnet is honest, the link between input and output is lost from the perspective of an observer.
Pedersen-style commitment

**Gen:** Select a subgroup $G \subset \mathbb{Z}_p^*$ of order $q$, and a generator $g$ of $G$. Choose $x \overset{R}{\leftarrow} Z_q$. Reveal $h = g^x$. 

Commit: To commit to a message $m \in G$, we choose $r \overset{R}{\leftarrow} Z_q$. The commitment is then:

$$c = g^m \cdot h^r.$$  

Open: To reveal the message $m$, the second component is sent:

$$d = g^r.$$ 

**Properties**

*Information theoretically hiding:* given the commitment $c$, any message $m' \in G$ is equally likely, and in particular, having the secret key $x$ one can compute:

$$r' = m - m' x + r.$$  

*Computationally binding:* finding two messages $m$ and $m'$ that open the commitment $c$ requires finding an $r$ and $r'$ s.t.

$$g^m \cdot h^r = g^{m'} \cdot h^{r'};$$  

then one can compute $\log_g(h) = m' - m r - r'$. 


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**Pedersen-style commitment**

**Gen:** Select a subgroup $G \subset \mathbb{Z}_p^*$ of order $q$, and a generator $g$ of $G$. Choose $x \xleftarrow{R} \mathbb{Z}_q$. Reveal $h = g^x$.

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**Properties**

**Information theoretically hiding:** given the commitment $c$, any message $m' \in G$ is equally likely, and in particular, having the secret key $x$ one can compute: $r' = \frac{m - m'}{x} + r$
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Properties

Information theoretically hiding: given the commitment $c$, any message $m' \in G$ is equally likely, and in particular, having the secret key $x$ one can compute: $r' = \frac{m-m'}{x} + r$

Computationally binding: finding two messages $m$ and $m'$ that open the commitment $c$ requires finding an $r$ and $r'$ s.t. $g^m \cdot h^r = g^{m'} \cdot h^{r'}$; then one can compute $\log_g(h) = \frac{m'-m}{r-r'}$. 
Selene: Voting in seven “easy” steps

1. Election Authority produces a tracker number $n_i$ and its encryption $e_i$ for each Voter $i$;

2. Mixnet shuffles the encrypted trackers $e_i$, resulting in a re-encryption $e'_i$ that loses connection to $n_i$;

3. Teller(s) generate Pedersen commitments $c_i$s for the $n_i$s, assign them to Voters $V_i$, then publish them to the Bulletin Board

4. Votes $v_i$ are encrypted ($e v_i$) and signed ($s_i$) by Voters $V_i$, and published along

5. Encrypted tracking numbers and votes $\langle e'_i, e v_i \rangle$ are shuffled by the Mixnet, then published as $\langle e''_i, e v'_i \rangle$, losing link to the originals;

6. Votes $e v_i$ and trackers $c_i$s are decrypted by the Tellers, and published to the Bulletin Board

7. Commitments are opened by the Teller(s) to the Voters, who can check that their vote has been casted.
To allow mechanised analysis in Tamarin, we assume an *external active adversary* who may collude with one of two voters.

Therefore:

- Only **one teller** is needed (since we assume it is honest)
- Re-encryption mixing is replaced by ballot shuffling
- **No zero-knowledge proofs** of secure computation are needed for the Teller, Bulletin Board and Election Authority
- We assume the existence of **authentic and confidential channels** for all communication between the honest parties
**Simplified Selene (1)**

**msc Generation of tracker numbers**

- **WBB**
  - Bulletin board

- **T**
  - Teller

- **EA**
  - Election authority

\[ \forall i \in \{1..max\}. \text{ choose } n_i \]

post \( n_i \)
Simplified Selene (3)

**msc Decryption**

- **WBB** (Bulletin board)
  - \( \forall i \in \{1..max\} \), read \( \langle n_i, ev_i \rangle \)
  - \( \langle n_i, ev_i \rangle \)
  - \( v_i = \text{decrypt } ev_i \)
  - \( \langle n_i, v_i \rangle \)
  - post \( \langle n_i, v_i \rangle \)

- **T** (Teller)

**msc Reveal and Check**

- **WBB** (Bulletin board)
  - \( b_i = g^{\tau_i} \)
  - read \( \langle pk_i, c_i \rangle \)
  - \( c_i \)
  - \( \langle n_i, v_i \rangle \)
  - \( n_i = \text{decrypt e}(b_i, c_i) \)

- **T** (Teller)

- **V_i** (Voter)

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Bruni et al, Proof of Selene Receipt-Freeness and Vote-Privacy
Theorem prover based on multiset-rewrite rules:

\[ l \rightarrow [\alpha] \rightarrow r \]

- States \( S \) are multiset of facts (initial state \( \emptyset \))
- Bang (\(!\)) modality for replicated facts
- Given a rule \( l \rightarrow [\alpha] \rightarrow r \) and a substitution \( \sigma \), a transition \( \sigma(l \rightarrow r) \) can fire on state \( S \) iff \( \sigma(l) \subseteq S \), and produces a state \( S' = S \setminus \alpha(l \uplus r) \)
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Observational equivalence “Given two systems, equivalent rules should fire in equivalent states.”

diff\((t_1, t_2)\) terms are used to distinguish the two systems
Rules:

- **Setup**, generates key pairs, initializes all agents
- **EA**, Election Authority, generates tracker numbers
- **T1**, teller commits a tracker number to each voter
- **V1**, voting phase
- **T2**, teller decrypts the encrypted vote
- **T2_{sync}**, all votes are shuffled
- **V2**, “reveal and check”
Equational theory

To model the commitment scheme we need the following two equations:

1. \( \text{open}(\text{commit}(n, r, pk(sk)), r, sk) = n \)

2. \( \text{commit}(n_2, \text{fake}(n_1, r, sk, n_2), pk(sk)) = \text{commit}(n_1, r, pk(sk)) \)
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Using Maude’s Church-Rosser checker we produce their Knuth-Bendix completion:

3. $open(commit(n_1, r, pk(sk)), fake(n_1, r, sk, n_2), sk) = n_2$
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However, this system of equations is still not confluent, to make it so we need to add the following equation:

4. $\text{fake}(n_2, \text{fake}(n_1, r, sk, n_2), sk, n_3) = \text{fake}(n_1, r, sk, n_3)$
Vote Privacy

- We build two models $S_L, S_R$ using diff terms where the two voters swap candidates.
- Tamarin proves that $S_L \approx_E S_R$
- We adapt the definitions from Delaune et al. 2008 to multiset-rewrite rules

Receipt Freeness

- We substitute rule $V2$ with two rules:
  1. One for the coerced voter, who reveals all his secret information, along with a *fake* or *real* opening of the commitment
  2. One for the colluding voter, who reveals his tracker number
Vote Privacy ✅

- As long as the voters are honest, the attacker cannot distinguish between the two systems.
Results

Vote Privacy ✓
- As long as the voters are honest, the attacker cannot distinguish between the two systems

Receipt Freeness ✗
- If the coerced voter hands a fake receipt for the tracker number of the colluding voter, the attacker can find out
- This attack is known (Ryan et. al 2016)
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