Risk-limiting audits for IRV elections

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October 5, 2018
Instant Runoff Voting (IRV)

- Preferential voting scheme
- A set of candidates $C$, one winner
- Each vote is a ranking over $C$
- Each vote can be a partial ranking
  e.g., [Mary Hill, Joe Smith, John Citizen]

Rank any number of options in your order of preference.

3  Joe Smith
2  John Citizen
    Jane Doe
    Fred Rubble
1  Mary Hill
Instant Runoff Voting (IRV) – An Example

4 candidates, 60 votes

<table>
<thead>
<tr>
<th>Ranking</th>
<th>Count</th>
</tr>
</thead>
<tbody>
<tr>
<td>[c₂, c₃]</td>
<td>4</td>
</tr>
<tr>
<td>[c₁]</td>
<td>20</td>
</tr>
<tr>
<td>[c₃, c₄]</td>
<td>9</td>
</tr>
<tr>
<td>[c₂, c₃, c₄]</td>
<td>6</td>
</tr>
<tr>
<td>[c₄, c₁, c₂]</td>
<td>15</td>
</tr>
<tr>
<td>[c₁, c₃]</td>
<td>6</td>
</tr>
</tbody>
</table>

(a) Initial tallies

<table>
<thead>
<tr>
<th>Candidate</th>
<th>Rnd1</th>
<th>Rnd2</th>
<th>Rnd3</th>
</tr>
</thead>
<tbody>
<tr>
<td>c₁</td>
<td>26</td>
<td>26</td>
<td>26</td>
</tr>
<tr>
<td>c₂</td>
<td>10</td>
<td>10</td>
<td>—</td>
</tr>
<tr>
<td>c₃</td>
<td>9</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>c₄</td>
<td>15</td>
<td>24</td>
<td>30</td>
</tr>
</tbody>
</table>

(b) Tallies after each round of counting
BRAVO Ballot-polling Risk Limiting Audits [LSY12], for first-past-the-post

- Given an announced election outcome (i.e. a set of winners with tallies), conduct a random audit of the paper ballots until either we’re confident the outcome is right, or we revert to a full manual recount.

- Choose a Risk Limit $\alpha$.

- Guarantee: If the outcome is wrong, we detect it with probability at least $1 - \alpha$.

- Works great for first-past-the-post, even with multiple winners, but what about IRV?
Maintain a running statistic $T_{wl}$ for each pair of apparent winner $w$ and loser $l$.

A ballot that shows a valid vote for winner $w$ increases $T_{wl}$ (by an amount dependent on the reported votes for the two candidates).

A ballot showing a valid vote for the loser $l$ decreases it.

When each statistic exceeds a threshold, dependent on the risk limit, we know that we have seen enough evidence to reject the hypothesis that $l$ beat $w$. 
Can we apply BRAVO to IRV?

* When you pick out a random IRV ballot, it’s not obvious whether it supports or undermines an announced election outcome.
First idea: audit the whole elimination sequence

- For every IRV elimination, run a BRAVO audit.
- The $k$-th elimination has $n - k - 1$ winners and 1 loser.
- Valid, but hideously inefficient.
- We don’t really need to know that the elimination order was right, only the final winner.
Optimisation 1: Batch eliminations

We can simultaneously eliminate candidates $E$ if the sum of tallies of these candidates is less than the tally of the next lowest candidate.

Audit using BRAVO by joining all the candidates in $E$ into one losing candidate.

Sometimes this is a huge improvement, e.g. if the differences between candidates in $E$ are small.

Sometimes it’s not.
We wish to eliminate the hypothesis that winner $w$ is eliminated before loser $l$.

1. **Winner** $w$ always has *at least* their first-preference votes.
2. **Loser** $l$ has *at most* the votes that mention $l$ somewhere, and don’t prefer $w$.
3. So run BRAVO with $T_{wl}$ counting case 1 as a vote for $w$ and case 2 as a vote for $l$.

If we can reject the hypothesis that $l$ beat $w$, that proves (except with probability at most $\alpha$) that $l$ was eliminated before $w$. 
Combining a series of facts to check that the announced winner is correct.

Confidence follows directly from BRAVO: if the announced outcome is wrong, one of the BRAVO audits will detect it with probability at least $1 - \alpha$.

Efficiency is heuristic. Often good, but no guarantees.

* e.g. (good): Berkeley Mayor 2012 (57,492 votes): Expect 115 samples for $\alpha = 0.05$.
  
* e.g. (bad): Gosford NSW 2015, Pierce County assessor 2008: no simulated audit smaller than recounting.
Questions?
M. Lindeman, P.B. Stark, and V. Yates. 
BRAVO: Ballot-polling risk-limiting audits to verify outcomes. 